Oscillating Elastic Defects: Competition and Frustration

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In his seminal 1957 paper, Eshelby derived the strain fields created by an inhomogeneous ellipsoidal inclusion in an isotropic elastic medium. This result is a cornerstone of the theory of inhomogeneous elastic media, now routinely used in the physical and engineering sciences. The result is a statement of how a distortion is accommodated in the host material and implies that a local perturbation induces long-range strain fields, slowly decaying as $1/r^d$ in d dimensions ($d \geq 2$). Eshelby's work considers static inhomogeneities or strain "defects" only. However, for many applications in physics and materials science, we are interested in the cooperative behavior of inhomogeneities in which the strain is varying or oscillating in time with a given frequency. We find that this situation is, as a function of the defect density and oscillation frequency, inherently frustrated, resulting in self-organization of the patterns, competing ground states, and sensitivity to internal and external perturbations. Such "dynamic" defects arise as small polarons in directionally-bonded transition metal oxides, including high-temperature superconductors, colossal magnetoresistance materials and ferroelectrics. The collective behavior of these polarons in a crystal undergoing distortions, with their coupling to charge, spin or polarization, is believed to determine the overall macroscopic response. Our work also has ramifications for the nondestructive evaluation of elastic media. Methods in this field are typically based on the vibrational response from a defect-free crystal. Here we characterize the behavior of oscillating defects (external oscillatory fields inducing specific defect patterns and responses) extending the conventional analysis to describe the response of elastic media in the presence of such defects [1].

The dynamical generalization of the Eshelby problem has been previously investigated in the context of engineering sciences for spherical inclusions, and very recently for inclusions of various shapes. However, such studies have focused on evaluating displacement fields as solutions to numerical boundary value problems. Our objective here is to understand the effect of dynamics on the nature of the elastic interaction itself and its influence on the collective behavior of assemblies of defects, using our recently developed elasticity formalism, which allows for tractable analytic calculations. We consider the dynamical Eshelby problem for localized oscillating defects in two dimensions for simplicity, and show that, although the strain fields still decay as $1/r^2$ far from the defect, the frequency fundamentally affects the nature of deformation. As expected, the higher the frequency, the more localized is the deformation. This renders the interaction between two defects strongly frequency (and direction) dependent, but the very nature of the interaction changes from "ferromagnetic" to "antiferromagnetic" like behavior as a function of separation and frequency. We subsequently generalize our results to a finite density of defects. This allows us to demonstrate the implications for frequency-driven patterning transitions and phase locking in assemblies of defects by mapping the elastic interaction energy between defects into XY spin-like models with competing interactions.

We use a strain only representation. The state of strain is defined by three fields e_i related to the displacements along the x-axis (u) and the y-axis (v) as follows: $e_1 = (u_x + v_y)/\sqrt{2}$, $e_2 = (u_y + v_x)/\sqrt{2}$, $e_3 = (u_x - v_y)/\sqrt{2}$ (the subscripts x and y indicate differentiation). We write an elastic energy which also includes strain gradient terms and the elastic compatibility constraint through a Lagrange multiplier. We assume for specificity that the oscillation is in e_3. We consider three different cases: one defect, two defects, and an assembly of oscillating defects with oscillation frequency ω_0 and phase ϕ_i. We use periodic boundary conditions in space and study overdamped as well as underdamped dynamics. In general, the larger ω_0, the more localized is the deformation created by the defect. This observation is of primary importance for the interactions between defects.

The main results are illustrated in Figs. 1 and 2, showing strain profiles for different ω_0. A comment on the underdamped case is in order: an underdamped dynamics would not remove the discontinuity at the wave vector $\bar{k} = 0$ created by the compatibility equation;

thus, the $1/r^2$ decay is also valid in this case. The qualitative effect of increasing ω_0 would not be modified either, although there would be some quantitative differences from the overdamped case. Figure 1 depicts the profile of the strain field e_3 for frequencies $\omega_0 = 1$ (diamonds), 10 (circles), and 100 (dots). The smaller ω_0, the wider the profile. The inset shows the surface plot of e_3: notice the anisotropy of the field. Figure 2 shows the Log-Log plot of the strain field e_3 along the diagonal for $\omega_0 = 1$, 10, and 100. We have also added $1/r^2$ fits as guides to the eye.

We have focused here on defects created by a locally oscillating e_3 strain; the solutions for locally oscillating e_1, e_2 strains, or combinations of the three strain components, can also be obtained, and are qualitatively similar. There is one exception to this statement: the solutions corresponding to area strain (e_1) are continuous around $\vec{k} = 0$ and even infinitely differentiable. Their tail in real space is thus *exponential* instead of *power law*. This implies that the e_1 strain field created by a local e_1 defect in an isotropic elastic medium decays exponentially away from the defect with rate $\rho(\omega_0)$; if the defect is oscillating, the exponential decay rate ω_0 grows with ω_0 as $\omega_0^{1/2}$. To our knowledge, this particular case has not been emphasized in the literature; it would be interesting to realize its experimental signatures.

We do not depict the results for two oscillating defects here. However, we note that for this case we find a more dramatic effect: varying the frequency, or the distance, can result in a qualitative change in the interaction, from "ferromagnetic" (phase-locking) to "antiferromagnetic" (anti-phase-locking). For N defects Fig. 3 shows the numerically determined boundary between "antiferromagnetic" ground states (when nearest neighbor interactions dominate) and more complicated ones, as expected from the two-defect calculations. The boundary between ferromagnetic and antiferromagnetic nearest neighbor interactions is also sketched. We have performed our calculations here for special arrangements of defects, but these results demonstrate the existence of collective behavior, controlled by the frequency or interparticle distance. The next natural step is to study phase-locking among defects by adding specific phase dynamics controlling the relaxation pathways. Finally, due to competing ground states we also expect multiscale "glassy" dynamics of oscillating defects.

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[1] J. Barré, et al., "Oscillating Elastic Defects," Los Alamos National Laboratory report LA-UR-05-4078 (May 2005); cond-mat/0508761.

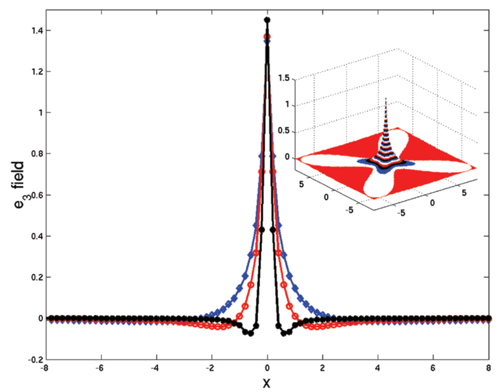


Fig. 1.
The profile of the strain field e_3 for frequencies $\omega_0 = 1$ (diamonds), 10 (circles), and 100 (dots). The smaller ω_0, the wider the profile. The inset shows the surface plot of e_3: notice the anisotropy of the field.

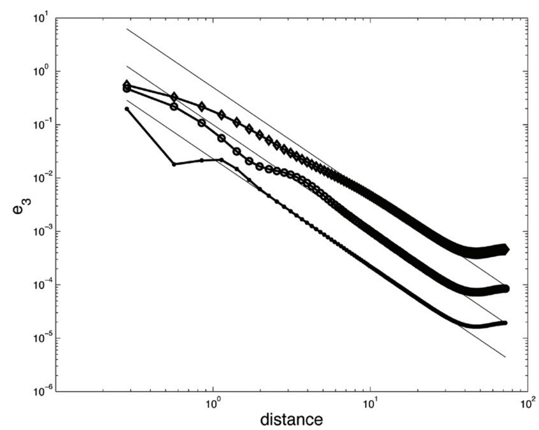
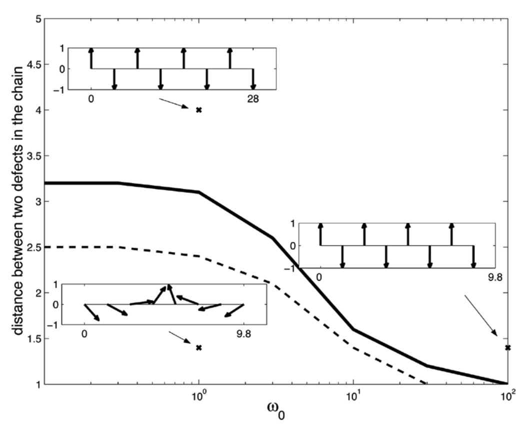


Fig. 2.
The Log-Log plot of the strain field e_3 along the diagonal for $\omega_0 = 1$, 10, and 100. We have also added $1/r^2$ fits as guides to the eye.



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The numerically determined boundary between "antiferromagnetic" ground states (when nearest neighbor interactions dominate) and more complicated ones, as expected from the two-defect calculations. The boundary between ferromagnetic and antiferromagnetic nearest neighbor interactions is also sketched.